

Disorder-enhanced phase coherence in trapped bosons on optical lattices

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The consequences of disorder on interacting bosons trapped in optical lattices are investigated by quantum Monte Carlo simulations. At small to moderate strengths of potential disorder a unique effect is observed: if there is a Mott plateau at the center of the trap in the clean limit, phase coherence *increases* as a result of disorder. The localization effects due to correlation and disorder compete against each other, resulting in a partial delocalization of the particles in the Mott region, which in turn leads to increased phase coherence. In the absence of a Mott plateau, this effect is absent. A detailed analysis of the uniform system without a trap shows that the disordered states participate in a Bose glass phase.

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Recent advances in experiments with ultracold atoms in magneto-optical traps have opened a new frontier in the study of strongly correlated systems. Some of the more remarkable early experimental breakthroughs include the realization of Bose-Einstein condensation[1] and fermionic superfluidity.[2] More recent achievements include tuning a degenerate Fermi gas across BCS-BEC crossover via Feshbach resonance,[3] and detecting superfluid-to-Mott insulator transitions of trapped bosons in optical lattices.[4, 5] An unprecedented control over experimental parameters in these systems makes them ideally suited for studying many-body phenomena.

Until recently, the majority of experiments with trapped ultracold atoms have been performed on clean systems. Indeed, the ability to create perfect (defect free) optical lattices is a major advantage over condensed matter experiments. On the other hand, the complete control over the trapping potentials makes it possible to introduce disorder and tune the disorder strength in a controlled fashion. The exciting possibilities of being able to study novel disorder-related phenomena such as Anderson localization and to explore novel quantum glassy phases have made the investigation of disorder in these systems an area of emerging interest.

Disorder can be generated in optical lattices by exposure to speckle lasers[6, 7], adding an incommensurate lattice-forming laser[8, 9], or other means[10]. It is thus possible to investigate different disorder-induced phenomena in a controllable manner, in contrast e.g. to previous studies with granular superconductors or ⁴He in vycor glass. The interplay between disorder and interactions in trapped Bose-Einstein condensates has recently been explored experimentally in ⁸⁷Rb, both in the continuum[7, 11, 12] and in an optical lattice[13]. On the theoretical front, such systems have been investigated within the frameworks of mean-field theories,[7, 8, 13, 14] Bose-Fermi mapping,[15] and the transfer matrix formalism.[16]

In this paper, we study interacting trapped bosons in one-dimensional optical lattices with a tunable random

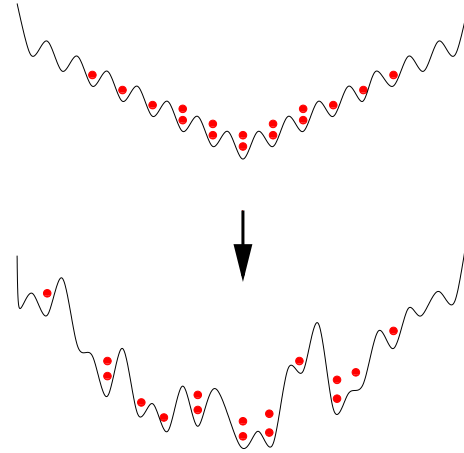


FIG. 1: Illustration of atoms in a potential trap. In the absence of interactions between the atoms, disorder leads to an immediate localization of the single-particle wave functions.

potential using a quantum Monte Carlo method. Previous investigations have argued that under realistic conditions, such diagonal disorder dominates over randomness in the hopping amplitude.[8] In experiments with optical lattices, the primary source of information about the state of the system arises from the analysis of the momentum distribution function, obtained from matter-wave interference after the release of the trap and subsequent free evolution of the particles. In view of this, here we focus on the signatures of disorder on the momentum distribution. The simulations presented here predict a unique enhancement of phase coherence at small to moderate strengths of disorder, when the groundstate has a Mott insulating region, i.e. a Mott plateau, at the center of the trap. Since Mott insulating states are now experimentally observable, and disorder strengths are controllable by tuning the intensity of the speckle laser, these results can be straightforwardly tested using currently available experimental methods. One technical difficulty in using current setups is the typically long

wavelength of speckle patterns (typically 8-10 times the optical lattice spacing) and the limited size of optical lattices (40-65 lattice spacings along each axis). However, these difficulties are likely to be overcome in the near future with advances in realizing larger lattices and / or using more than one speckle laser to create disordered potentials with a shorter correlation.

Bosons in an optical lattice are well described by the one-band Bose Hubbard model.[18] In the presence of a strong periodic (lattice) potential, single-particle Wannier functions localized on the lattice sites form a complete basis set. Interactions are typically not strong enough to excite higher vibrational bands, justifying the single band approximation. Hence the low-energy physics of trapped bosons in a one-dimensional optical lattice can be described by the Hamiltonian

$$H = -t \sum_{i=1}^L (b_{i+1}^\dagger b_i + h.c.) + \frac{U}{2} \sum_{i=1}^L n_i(n_i - 1) - \mu_0 \sum_{i=1}^L n_i + V_T a^2 \sum_{i=1}^L (i - \frac{L}{2})^2 n_i + \sum_{i=1}^L W_i n_i. \quad (1)$$

Here $b_i^\dagger (b_i)$ creates (annihilates) a boson at site i , $n_i = b_i^\dagger b_i$ is the number operator, and U is the strength of the repulsive on-site Hubbard interaction between bosons. The bare chemical potential μ_0 controls the filling of the lattice, V_T is the strength of the trapping potential, a denotes the lattice spacing, and W_i introduces diagonal disorder in the form of a random site energy. The hopping amplitude t is set equal to unity in the simulations, and all other parameters are implicitly expressed in units of t . In the experiments, W_i is commonly realized by a speckle laser. Furthermore, recent measurements indicate significantly correlated disorder, with a correlation length that is typically several times longer than the optical lattice spacing, due to the diffraction-limited imaging of the speckles coming from a diffusion plate onto the trapped atoms.[11, 12] In view of this, we consider finitely correlated disorder with a random potential W_i that is distributed as

$$\overline{W_i} = 0, \quad \overline{W_i W_{i'}} = \Delta \delta_{i,i'}. \quad (2)$$

The overbar denotes averaging over disorder realizations, and Δ is disorder strength. The indices l are measured in units of $5a$.

In the absence of disorder, the Bose Hubbard model (1) in a trap has been extensively studied.[19] Due to the spatially varying trapping potential, the system is never in a uniform phase. Depending on the strength of the trapping potential, V_T , the on-site interaction, U , and the density of particles, n , the groundstate can belong to one of two classes. At sufficiently small U/t and total density, n , it consists of a superfluid (SF) domain extending across the entire system. The site-occupation,

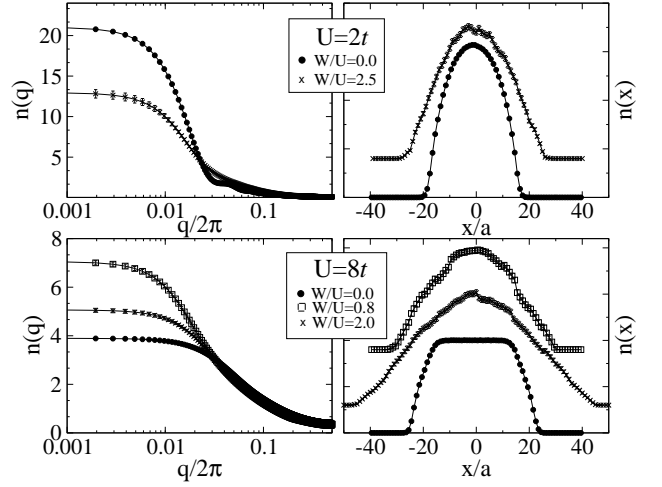


FIG. 2: Effects of disorder on the momentum distribution $n(q)$ and on the real-space density profile $n(x)$ in the weak (upper panels) and strong (lower panels) coupling limit. The density profiles are vertically offset to enhance clarity of presentation.

n_i varies continuously from zero at the edges to a finite maximum value at the center of the trap. The momentum distribution, $n(q)$, features a sharp peak at $q = 0$ reflecting strong phase coherence across the entire system. The full width at half maximum of the zero-momentum peak is inversely proportional to the correlation length ξ . In a thermodynamic SF ξ diverges, whereas in the one-dimensional confined SF $\xi \propto N$ and $n(q=0) \propto \sqrt{N}$.

On the other hand, at larger U/t and sufficiently large n , the groundstate contains one or more Mott insulating (MI) regions with integer n_i , along with domains of SF at the edges of the trap and in between the MI regions (if there are more than one such regions).[19] In the present study, we consider the simplest case of one MI plateau at the center of the trap and two SF domains at the edges – the local density profile consists of an extended region at the center with $n_i = 1$ and two regions with $0 < n_i < 1$ near the trap edges. In the presence of MI region(s), the range of coherence is greatly reduced, and the momentum distribution shows a weak peak at $q = 0$, and a subsequent slow decrease in $n(q)$ as a function of q . Depending on whether there exists an MI region at the center of the trap, we find the effects of disorder can be markedly different.

We use the stochastic series expansion quantum Monte Carlo method to simulate the disordered Bose-Hubbard model on finite-sized lattices.[17] The density profiles, $n(x)$, and the momentum distribution function, $n(q)$, are measured within the grand canonical ensemble, using sufficiently large values of the inverse temperature, β , in order to obtain groundstate properties. Disorder averages over 400-2000 realizations are performed, depending on the parameters. The lattice sizes are chosen to be suffi-

cently large to ensure that the local density vanishes at the edge of the trap. We found that a lattice size $L = 120$ is necessary for large values of $U (\geq 8t)$ at strong disorder ($W/U > 1.5$), whereas $L = 80$ was sufficient for the other parameter sets. Results are displayed for 40 bosons in a parabolic trap with $V_T a^2 = 0.015t$ for a range of values of U/t , ranging from the weak to strong coupling limit.

Fig. 2 shows the consequences of a disordered site-potential on the momentum distribution function $n(q)$ and the real-space density profile $n(x)$ for two values of the on-site interaction, U/t . The upper panels show the results for $U/t = 2$, when the groundstate in the clean limit ($W = 0$) consists of a single SF domain extending across the trap. Adding disorder produces localization centers around which the wave functions get localized. This in turn reduces coherence and hence results in a smaller zero-momentum peak. At sufficiently weak disorder strengths, these effects are found to be relatively small in finite systems, and the groundstate retains a predominantly SF character – the deviation of the momentum distribution from its clean limit is exponentially small. However, with increasing disorder strength, the localization effects strengthen, and hence the momentum distribution starts to deviate significantly, i.e. the peak at $n(q \rightarrow 0)$ becomes shorter and broadened.

On the other hand, for $U/t = 8$ (lower panels), the groundstate of the system, in the clean limit, has a Mott plateau at the center and the effects of disorder are considerably more interesting. The wave functions in the MI region in the absence of disorder are localized due to the strong Hubbard interactions. The onset of disorder weakens the correlation between particles, and thus leads to a partial delocalization of the strongly localized Wannier orbitals at small to moderate strengths of disorder. Phase coherence hence *increases* as the MI region “melts”, reflected in a stronger $q = 0$ peak in $n(q)$ with increasing disorder. For larger disorder potentials, the Anderson localization effects dominate, and the zero-momentum peak starts to decrease.

Fig. 3 shows the variation of the strength of the zero-momentum peak as a function of disorder for four different values of U/t , ranging from the weak to the strong coupling limit. For intermediate values of the interaction strength, the peak at $n(q = 0)$ increases at small to intermediate disorder (reflecting the melting of the central MI region), reaches a maximum, and finally decreases continuously for larger W . This is to be contrasted with the weak coupling limit, where $n(q = 0)$ remains fairly unchanged for small W , and then decreases rapidly at larger disorder strengths. The exact position of the maximum and its value, $n_{max}(q = 0)$ depends on U/t , V_T/t , and n . The peak position shifts to larger disorder strengths with increasing Hubbard interaction strength, indicating a competition between Mott localization and Anderson localization that depends on the ratio of W vs. U . The maximum height, $n_{max}(q = 0)$ and its ratio to the clean

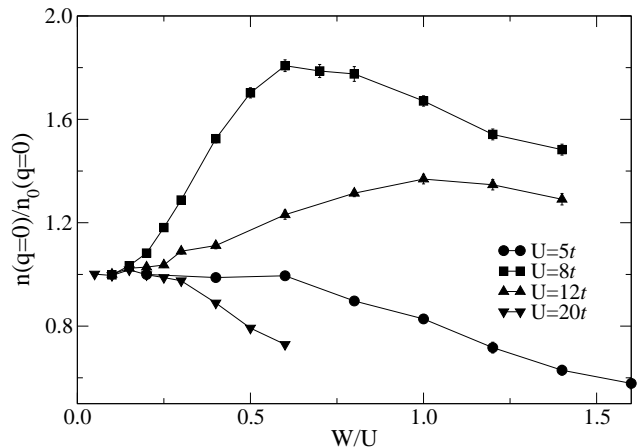


FIG. 3: Dependence of the zero-momentum peak height, $n(q = 0)$, on the disorder strength W for different values of U/t . At $U/t = 5$, the groundstate consists of a single SF domain, whereas for the other values, there exist a finite MI region at the center of the trap.

system value, $n_0(q = 0)$, decrease rapidly with U/t . At $U/t = 20$, i.e. close to the hard-core limit, no visible increase with W is noticeable in $n(q = 0)$.

What is the nature of the state that results from the onset of disorder? Does the partial delocalization of the MI states result in a SF or a glassy phase? To answer these questions, we analyze the effects of disorder in the absence of a trapping potential. At $V_T = 0$, the groundstate is in a pure phase, and can be conveniently characterized by measuring *global* observables, such as the compressibility and the stiffness. For shallow trapping potentials, the system is adiabatically connected to the periodic case, such that the domains of the different phases retain their global characteristics to a large extent. This allows us to study the effects of disorder in the pure phases and extend the results to corresponding domains in the trapped system. Let us focus on the global compressibility, $\kappa = \beta(\langle n^2 \rangle - \langle n \rangle^2)$, and the stiffness, ρ_s , to differentiate between various possible phases – SF, MI or Bose glass (BG). The stiffness gives the response of the system to a uniform twist in the spatial boundary as $\rho_s = \partial^2 E / \partial \phi^2$, under which the kinetic energy term in the Hamiltonian (1) is replaced by $-t \sum (e^{-i\phi} b_{i+1}^\dagger b_i + h.c.)$. In practice, the stiffness is simply related to the fluctuations in the winding number in the simulations as $\rho_s = \langle W^2 \rangle / 2\beta L$. [20] The results shown in Fig. 4 are for uniform SF (left panels) and MI (right panels) phases in the presence of potential disorder. The dependence of $n(q = 0)$ on W is qualitatively similar compared with the case of trapped atoms. For the SF phase, $n(q = 0)$ remains practically unchanged at small W , and starts to decrease monotonically for larger disorder. On the other hand, in the MI phase $n(q = 0)$ first increases and then decreases with increasing W , confirm-

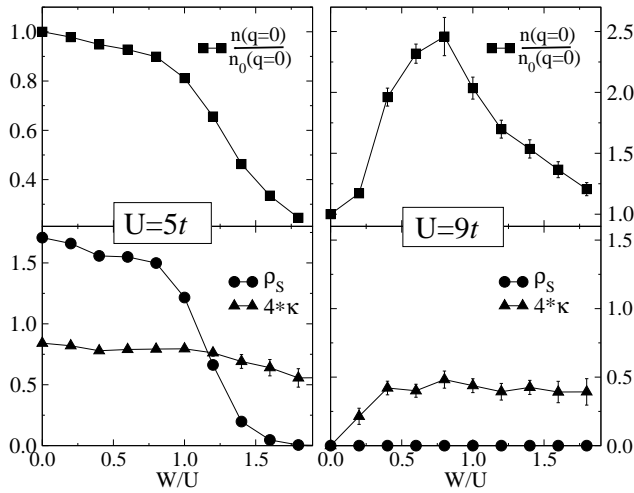


FIG. 4: Dependence of the momentum distribution function at $q = 0$, the stiffness and the compressibility of SF (left panels) and MI (right panels) phases on disorder strength. For the SF phase, the groundstate is predominantly SF at small to moderate disorder and a Bose glass at large disorder. For the MI phase, the groundstate is a Bose glass at all strengths of disorder studied. The strength of the zero-momentum peak varies non-monotonically, but the groundstate always remains a Bose glass.

ing that the disordered states are qualitatively similar to their counterparts in the presence of a trapping potential. The stiffness and compressibility data allow us to further characterize the disordered states. For the SF groundstate, both ρ_s and κ are finite for the range of W over which $n(q = 0)$ remains close to its $W = 0$ value, consistent with a predominantly SF character of the disordered state. At larger W , as $n(q = 0)$ starts to deviate significantly, the compressibility remains finite, but the stiffness rapidly decreases to zero. Hence the groundstate at large disorder is a compressible insulator, i.e. a *Bose glass* (BG). From the present data it is not clear if one needs a finite critical disorder to destroy superfluidity or simply the localization length at small disorder is larger than the system sizes considered here. For the MI phase, both ρ_s and κ vanish in the clean system. With the onset of disorder, the stiffness remains zero, but the compressibility acquires a finite value. The disordered MI is thus a BG at all strengths of disordered considered here.

In summary, based on quantum Monte Carlo simulations ensembles of interacting trapped atoms are found to respond in a highly non-trivial fashion to diagonal disorder. If the clean system consists of a single superfluid domain, the particles are simply localized by the random potential. This is reflected by progressive suppression of the $q \rightarrow 0$ peak in the momentum distribution function. On the other hand, if the clean system contains Mott insulating regions the response to on-site disorder is non-monotonic, and depends on the relative strength of the Hubbard interaction and the disorder potential. Be-

cause of this competition there is an intermediate regime with enhanced phase coherence. An analysis of the compressibility and stiffness in the corresponding periodic systems, i.e. in the absence of a trapping potential, reveals that the resulting disordered state in either case is a Bose glass. The predicted enhancement of phase coherence by diagonal disorder suggests an interesting extension of recent experiments on optical traps in the presence of a tunable speckle laser.[12]

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- [1] M. H. Anderson *et al.*, Science **269**, 198 (1995).
 - [2] B. DeMarco, and D. S. Jin, Science **285**, 1703 (1999).
 - [3] K. M. O'Hara, *et al.*, Science **298**, 2179 (2002); M. Greiner, *et al.*, Nature **426**, 537 (2003); S. Jochim, *et al.*, Science **302**, 2101 (2003).
 - [4] M. Greiner, *et al.*, Nature **415**, 39 (2002).
 - [5] T. Stöferle, *et al.*, Phys. Rev. Lett. **92**, 130403 (2004).
 - [6] P. Horak, J. -Y. Courtois, and G. Grynberg, Phys. Rev. A **58**, 3953 (2000).
 - [7] D. Clément, A. F. Varón, M. Hugbart, J. A. Retter, P. Bouyer, L. Sanchez-Palencia, D. M. Gangardt, G. V. Shlyapnikov, and A. Aspect, Phys. Rev. Lett., **95**, 170409 (2005).
 - [8] B. Damski, J. Zakrzewski, L. Santos, P. Zoller, and M. Lewenstein, Phys. Rev. Lett. **91**, 080403 (2003).
 - [9] A. Sanpera, A. Kantian, L. Sanchez-Palencia, J. Zakrzewski, and M. Lewenstein, Phys. Rev. Lett., **93**, 040401 (2004).
 - [10] R. Folman, P. Krüger, J. Schmiedmayer, J. Denschlag, and C. Henkel, Adv. At. Mol. Opt. Phys. **48**, 263 (2002).
 - [11] J. E. Lye, L. Fallani, M. Modugno, D. S. Wiersma, C. Fort, and M. Inguscio, Phys. Rev. Lett. **95**, 070401 (2005).
 - [12] C. Fort, L. Fallani, V. Guarrera, J. E. Lye, M. Modugno, D. S. Wiersma, and M. Inguscio, Phys. Rev. Lett. **95**, 170410 (2005).
 - [13] T. Schulte, S. Drenkelforth, J. Kruse, W. Ertmer, J. Arlt, K. Sacha, J. Zakrzewski, and M. Lewenstein, Phys. Rev. Lett. **95**, 170411 (2005).
 - [14] R. C. Kuhn, C. Miniatura, D. Delande, O. Sigwarth, and C. A. Müller, cond-mat/0506371 (unpublished).
 - [15] A. De Martino, M. Thorwart, R. Egger, and R. Graham, Phys. Rev. Lett., **94**, 060402 (2005).
 - [16] U. Gavish, and Y. Castin, Phys. Rev. Lett., **95**, 020401 (2005).
 - [17] A. W. Sandvik, Phys. Rev. B **59**, R14157 (1999).
 - [18] D. Jaksch, C. Bruder, J.I. Cirac, C.W. Gardiner, and P. Zoller, Phys. Rev. Lett. **81**, 3108 (1998).
 - [19] G. G. Batrouni, *et al.*, Phys. Rev. Lett. **89**, 117203 (2002); S. Wessel, F. Alet, M. Troyer, and G. G. Batrouni, Phys. Rev. A, **70**, 053615 (2004); P. Sengupta, M. Rigol, G. G. Batrouni, P. J. H. Denteneer, and R. T. Scalettar, Phys. Rev. Lett. **95**, 220402 (2005).
 - [20] E.L. Pollock and D.M. Ceperly, Phys. Rev. B **36**, 8343

- (1987).
- [21] R. M. Kalas, and D. Blume, cond-mat/051203 (unpublished).